and this exchange is manifested in the energy supply, $\underset{\sigma}{\hat{\epsilon}_\alpha}$. In the absence of heat conduction and energy transfer (i.e. $h_\alpha^- = 0$, $\underset{\sigma}{\hat{\epsilon}_\sigma} = 0$, $\alpha = 1, 2, \ldots, k$), Equations (51) and (52) combine to yield a particularly simple relation between the total energy density for any constituent and that for the whole mixture.

$$c_\alpha^- \left(\epsilon_\alpha^- + \frac{1}{2} (v^-)^2 \right) = n_\alpha^- \left(\epsilon^- + \frac{1}{2} (v^-)^2 \right) \tag{53}$$

Equations (3) and (17), in addition to (51) and (52), have been used to obtain this relation.

CONCLUSIONS

The simplified theory presented in the preceeding section can now be completed by the addition of certain constitutive relations. The resulting set of coupled algebraic equations may then be solved to obtain any of the usual Hugoniot descriptions ($P^- - \eta$, $P^- - v^-$, $U - v^-$, etc.) for the whole mixture. In this manner, the Hugoniot for a composite material may be constructed from known equation-of-state data for its constituents, plus constitutive relations governing mass and energy transfer, and heat conduction.

The simplest possible theory occurs when none of the mixture constituents is heat conducting, and no mass or energy transfer occurs. That is

$$h_\alpha^- = \underset{\sigma}{\hat{c}_\alpha} = \underset{\sigma}{\hat{\epsilon}_\alpha} = 0, \qquad \alpha = 1, 2, \ldots, k \tag{54}$$

This case is equivalent to the adiabatic theory of Tsou and Chou [3]. No heat is transferred between constituents, and phase transformations cannot be considered.

The following equations result from the assumptions of Equation (54). Jump relations for mass, momentum, and energy for the whole mixture are

$$\rho^-(U - v^-) = \rho_0 U \tag{55}$$

$$\widetilde{P}^- = \rho^-(U - v^-) v^- \tag{48}$$

$$\widetilde{P}^- v^- = \rho^-(U - v^-) \left(\epsilon^- + \frac{1}{2} (v^-)^2 \right) \tag{56}$$

These three equations are then augmented by the energy balance relation for each of the constituents obtained from Equation (53).

$$c_{\alpha_0} \left(\epsilon_\alpha^- + \frac{1}{2} (v^-)^2 \right) = n_\alpha^- \left(\epsilon^- + \frac{1}{2} (v^-)^2 \right), \qquad \alpha = 1, 2, \ldots, k \tag{57}$$

Note that, when no mass transfer occurs, the concentration, c_α^-, following the shock must equal the initial concentration, c_{α_0} [Equation (50)]. The volume fraction, n_α^-, can be related to the constituent crystal density, $\tilde{\rho}_\alpha^-$, by combining Equations (3) and (17). Thus

$$n_\alpha^- = \frac{c_{\alpha_0}\rho^-}{\tilde{\rho}_\alpha^-}, \qquad \alpha = 1, 2, \ldots k \tag{58}$$

while Equation (20) relates the mixture and constituent densities

$$\frac{1}{\rho^-} = \sum_\alpha \frac{c_{\alpha_0}}{\tilde{\rho}_\alpha^-} \tag{59}$$

Finally, we have the equation-of-state for each of the constituents.

$$\widetilde{P}^- = F_\alpha(\tilde{\rho}_\beta^-, \epsilon_\beta^-), \qquad \alpha, \beta = 1, 2, \ldots, k \tag{60}$$

Equations (55) through (60) now constitute $3k + 4$ equations in $3k + 5$ unknowns: ρ^-, U, v^-, $\widetilde{P}^-$, ϵ^-, n_α^-, $\tilde{\rho}_\alpha^-$, ϵ_α^-, $\alpha = 1, 2, \ldots, k$. As in any Hugoniot description, one variable is specified in order to solve for all others.

This specialized theory may now be expanded by eliminating any of the three assumptions of Equation (54). For example, if one wishes to consider heat conduction within the constituents, it is necessary to assume constitutive relations for the fluxes, $\tilde{h}_\alpha^-$, and to replace Equation (57) by

$$\tilde{\rho}_\alpha^-(U - v^-)\left(\epsilon_\alpha^- + \frac{1}{2}(v^-)^2\right) + \tilde{h}_\alpha^- = \rho^-\left(\epsilon^- + \frac{1}{2}(v^-)^2\right) + h^- \tag{61}$$

subject to Equation (32). On the other hand, if energy transfer among constituents is desired, Equation (57) is replaced by

$$c_{\alpha_0}\left(\epsilon_\alpha^- + \frac{1}{2}(v^-)^2\right) = n_\alpha^-\left(\epsilon^- + \frac{1}{2}(v^-)^2\right) + \frac{\underset{\sigma}{\hat{\epsilon}}_\alpha}{\rho_0 U} \tag{62}$$

Then it is necessary to postulate constitutive equations for the energy supply terms, $\underset{\sigma}{\hat{\epsilon}}_\alpha$.

$$\underset{\sigma}{\hat{\epsilon}}_\alpha = E_\alpha(\tilde{\rho}_\beta^-, \epsilon_\beta^-), \qquad \alpha, \beta = 1, 2, \ldots, k \tag{63}$$

subject to Equation (15). Finally, if mass transfer is to be considered, Equation (57) is replaced by Equation (53). Equations (20) and (50) are also employed and constitutive relations of the form

$$\underset{\sigma}{\hat{c}}_\alpha = C_\alpha(\tilde{\rho}_\beta^-, \epsilon_\beta^-), \qquad \alpha, \beta = 1, 2, \ldots, k \tag{64}$$

can be assumed. Any two, or all three, of the assumptions in Equation (54) may be easily dropped by combining the equations noted above.

In application to a particular composite material, the constitutive relations [Equations (63) and (64)] may be formulated in any manner consistent